
SYSTEMATIZE LEVEL INVENTORY MODEL FOR DETERIORATING ITEMS

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ABSTRACT

In the year 1950, the Operations Research Society of America was formed in order to increase the impact of OR and to established the rapport between its students and users. In 1957, the International Federation of OR Societies was established. In several countries, International Scientific Journals in Operations Research began to appear in different languages. The primary Journals are Operations Research, Transportation Science, Management Science Operational Research Quarterly, Journal of Canadian Operational Research Societies etc.

Key words: Operation Research, International Federation

INVENTORY

Inventory is simply a stock of physical assets having some economic value, which can be either in the form of material, money or labor. Inventory is also known as an idle resource as long as it is not utilized. Inventory may be regarded as those goods, which are produced, stored and used for day today functioning of the organization. This can be in the form of physical resource such as raw materials, semi-finished goods used in the production process, finished products ready for delivery to consumers; human resources such as an utilized labor or financial resource such as working capital etc. Inventory is any stored resource that is used to satisfy a current or a future need. Raw materials, work in process and finished goods are examples of inventory. Inventory levels for finished goods are direct functions of demand. When determine the demand for completed clothes, dryers; for example, it is possible to use this information to determine how much sheet metal, paint, electric motors, switches and other raw materials and work in process are needed to produce the finished product. Inventories constitute the largest component of current assets in many organizations. Poor management of inventories therefore may results in business failures. A stock-out creates an unpleasant situation for the organization. In case of a manufacturing organization, the inability to supply an item from inventory could bring production process to a halt. Conversely, if a firm carries excessive inventories, the added carrying cost may represent the difference between profit and loss. Efficient inventory control, therefore, can significantly contribute to the overall profit-position of the organization.

INTRODUCTION

Classical inventory models developed for constant demand rate can be applied to both manufacturing and sales environment. But many super market managers have observed that, for some items, the demand rate is directly related to the amount of inventory displayed. Some modelers have taken constant deterioration rate in their inventory models. In practice it can be observed that constant rate of deterioration occurs rarely. Most of the items deteriorate fast as the time passes. It can be noticed that deterioration does not depends upon time only. It can affect due to weather conditions, humidity, storage conditions etc. The success of business tactics has close relationship with controlling inventory. Too much inventory reduces capital and increases storage expenses and risk. In the existing literature of inventory control, the conventional inventory models have been developed under the assumption that the life time of an item is infinite during their storage period. This means that an item once in stock remains unchanged and fully usable for satisfying the future demand in a perfect condition. In reality, this assumption is not always true for some commonly used physical goods like wheat, paddy or any other type of food grains, medicines, vegetables, fruits, etc. due to their deterioration effect. A certain fraction of these goods are either damaged or decayed by some other factors. Hence, deterioration effect is an important factor to the inventory management.

Determining the correct inventory level at the right time and reducing cost and satisfying customer demand are the most important issues for the managers. To avoid or to decrease the loss due to deterioration effect and demand may need to be backlogged for highly perishable goods and these can be procured in an economic manner. In many situations, the customers of certain suppliers have high faith and loyalty. When there is a shortage, these customers are willing to wait for backorders. However, if the customer becomes impatient and becomes turns to other suppliers lost sales will result. This situation is modeled by the consideration of partial backordering in the formulation of mathematical models.

REVIEW OF LITERATURE

Therefore it is much more realistic to consider deterioration rate as two or three parameter Weibull distribution functions. In this section, two inventory models are developed with deterioration rates as two parameter and three parameter Weibull distribution function of time. Demand for these models is observed to be exponentially increasing for some initial period. Giri and Chaudhari (1998), Sana et al (2004), Chung (2000), Teng, Chang and Goyal (2005), Khanra and Chaudhari (2003), Chang and Dye (1999), Papachristos and Skouri (2000), Aggarwal and Jain (2001) and Kumar and Sharma (2000) have used constant deterioration in their inventory models. But the concept of constant deterioration is not realistic in real life situations. Therefore in our inventory model we have taken variable deterioration rate. Further more, when the shortages, occur, some customers are willing to wait for backorder and other would turn to by from other sellers. Many researchers such as Park (1982), Hollier and Mak (1983) and Wee (1995) considered the constant partial backlogging rates during the shortage period in their inventory models. In some inventory systems, such as fashionable commodities, the length of the waiting time for the next replenishment would determine whether the backlogging would be accepted or not. Therefore, the backlogging rate is variable and dependent on the length of the waiting time for the next replenishment. Abad (1996) investigated an EOQ model allowing shortage and partial backlogging. Some researchers Balkhi and Benkherouf (2009), Chu and Chung (2010) and Giri and Yun (2011) developed inventory model with constant demand rate. But the concept of constant demand rate is not so realistic therefore we have taken variable demand in our model.

In this model, we have taken deterioration rate as linear function of time, demand rate exponentially increasing of time and production rate is dependent on demand rate. Shortages are allowed and partially backlogged and backlogging rate is dependent on waiting time for next replenishment.

ASSUMPTIONS:

The production inventory model for deteriorating items is developed on the basis of the following assumptions and notations:

- (i) $I(t)$ is the inventory level at any time $t \geq 0$.
- (ii) The demand rate $R(t) = ae^{-bt}$ where a and b are the positive constant and $0 < b \ll 1$.
- (iii) The replenishment rates $K = rR(t)$ where $r > 1$ a constant.
- (iv) A fraction $Z(t) = \theta t$, $0 < \theta \ll 1$, $t > 0$, of the on hand inventory deteriorates per unit time.
- (v) The deteriorating units can be neither replaced nor repaired during the cycle time.
- (vi) The lead time is zero.
- (vii) C' , C_H , C_s , C_D , C_L denote the set up cost for each replenishment, inventory carrying cost per unit time, shortage cost for backlogged items, deteriorating cost per unit, the unit cost of lost sales respectively. All of the cost parameters are positive constants.
- (viii) Shortages are allowed and backlogging rate is taken as δ , where δ is the backlogging parameters and $0 < \delta < 1$.
- (ix) T is the time horizon.
- (x) A single item is considered over the prescribed period of time.

MATHEMATICAL MODEL AND ANALYSIS

Initially the inventory level is zero. The production starts at time $t = 0$ and after t_1 , units of time, it reaches to maximum inventory level. After this production stopped and at time $t = t_2$ the inventory level becomes zero. At this time shortage start developing at time $t = t_3$, it reaches to maximum shortages level. At this fresh production starts to clear the backlog by the time $t = T$ our aim is to find the optimum values of t_1, t_2, t_3, T that minimize. The total average cost C over the time horizon $(0, T)$.

Let $I(t)$ be the inventory level at any t , $0 \leq t \leq T$.

The governing differential equation of any inventory system in the interval $[0, t)$ are

$$\frac{dI(t)}{dt} + z(t)I(t) = K - R(t), \quad 0 \leq t \leq t_1 \quad \dots(1.1)$$

$$\frac{dI(t)}{dt} + z(t)I(t) = -R(t), \quad t_1 \leq t \leq t_2 \quad \dots(1.2)$$

$$\frac{dI(t)}{dt} = -\delta R(t), \quad t_2 \leq t \leq t_3 \quad \dots(1.3)$$

$$\frac{dI(t)}{dt} = K - \delta R(t), \quad t_3 \leq t \leq T \quad \dots(1.4)$$

With the condition, $I(0) = I(t_2) = I(T) = 0$. using the value of $R(t)$, $Z(t)$ and K the equation (1.1), (1.2), (1.3) and (1.4) can be written as

$$\frac{dI(t)}{dt} + \theta t I(t) = (r-1)ae^{-bt}, \quad 0 \leq t \leq t_1 \quad \dots(1.5)$$

$$\frac{dI(t)}{dt} + \theta t I(t) = -ae^{-bt}, \quad t_1 \leq t \leq t_2 \quad \dots(1.6)$$

$$\frac{dI(t)}{dt} = -\delta ae^{-bt}, \quad t_2 \leq t \leq t_3 \quad \dots(1.7)$$

$$\frac{dI(t)}{dt} = (r-\delta).ae^{-bt}, \quad t_3 \leq t \leq T \quad \dots(1.8)$$

The solution of equation (1.5) is given by

$$I(t)e^{\frac{\theta t^2}{2}} = a(r-1) \int e^{-bt} \cdot e^{\frac{\theta t^2}{2}} dt + A_1$$

where A_1 is the constant of integration.

$$I(t) = A_1 e^{-\frac{\theta t^2}{2}} + a(r-1) \left[t - \frac{bt^2}{2} - \frac{\theta t^3}{3} - \frac{b\theta t^4}{4} \right]$$

$$I(t) = A_1 \left(1 - \frac{\theta t^2}{2} \right) + a(r-1) \left[t - \frac{bt^2}{2} - \frac{\theta t^3}{3} - \frac{b\theta t^4}{4} \right]$$

But $I(0) = 0$, therefore $A_1 = 0$

$$I(t) = a(r-1) \left[t - \frac{bt^2}{2} - \frac{\theta t^3}{3} + \frac{b\theta t^4}{4} \right], \quad 0 \leq t \leq t_1 \quad \dots(1.9)$$

The solution of equation (1.6) is given by

$$I(t) \cdot e^{\frac{\theta t^2}{2}} = -a \int e^{\frac{\theta t^2}{2}} \cdot e^{-bt} dt + A_2$$

where A_2 is the constant of integration.

$$I(t) = A_2 \cdot e^{-\frac{\theta t^2}{2}} - a \left[t + \frac{\theta t^3}{3} - \frac{bt^2}{2} \right] e^{-\frac{\theta t^2}{2}}$$

But $I(t_2) = 0$, therefore

$$A_2 = a \left[t_2 + \frac{\theta t_2^3}{3} - \frac{bt_2^2}{2} \right]$$

$$I(t) = a \left[t_2 - \frac{bt_2^2}{2} + \frac{\theta t_2^3}{2} \right] e^{-\frac{\theta t^2}{2}} - a \left[t + \frac{\theta t^3}{3} - \frac{bt^2}{2} \right] e^{-\frac{\theta t^2}{2}}$$

$$I(t) = \left[(t^2 - t) - \frac{b}{2}(t_2^2 - t^2) - \frac{\theta t_2 t^2}{2} + \frac{b\theta t_2^2 t^2}{2} + \frac{\theta t_2^3}{3} + \frac{\theta t^3}{6} \right], \quad t_1 \leq t \leq t_2 \quad \dots(1.10)$$

The solution of equation (1.7) is given by

$$I(t) = \frac{\delta a}{b} e^{-bt} + A_3$$

where A_3 is the constant of integration.

But $I(t_2) = 0$

$$A_3 = -\frac{\delta a}{b} e^{-bt_2}$$

$$I(t) = \frac{\delta a}{b} [e^{-bt} - e^{-bt_2}], \quad t_2 \leq t \leq t_3 \quad \dots(1.11)$$

The solution of equation (1.8) is given by

$$I(t) = \frac{-a(r-\delta)}{b} e^{-bt} + A_4$$

where A_4 is the constant of integration.

But $I(T) = 0$, therefore

$$A_4 = \frac{a(r-\delta)}{b} e^{-bT}$$

$$I(t) = \frac{a(r-\delta)}{b} [e^{-bT} - e^{-bt}], \quad t_3 \leq t \leq T \quad \dots(1.12)$$

Total number of unit holding is given by

$$I_H = \int_0^{t_1} I(t) dt + \int_{t_1}^{t_2} I(t) dt = a(r-1) \left[\frac{t}{2} - \frac{bt^3}{6} - \frac{\theta t^4}{12} + \frac{b\theta t^5}{20} \right]_0^{t_1} + a \left[\left(t_2 t - \frac{t^2}{2} \right) - \frac{b}{2} \left(t_2^2 t - \frac{t^3}{3} \right) \right. \\ \left. - \frac{\theta t_2 t^3}{6} + \frac{b\theta t_2^2 t^3}{6} + \frac{\theta t_2^3 t}{3} + \frac{\theta t^4}{2} \right]_{t_1}^{t_2}$$

$$I_H = \left[r \left\{ \frac{t_1^2}{2} - \frac{bt_1^3}{6} - \frac{\theta t_1^4}{12} + \frac{b\theta t_1^5}{20} \right\} + \frac{t_2^2}{2} - \frac{bt_2^3}{6} + \frac{5\theta t_2^4}{24} + \frac{b\theta t_2^5}{6} \right. \\ \left. - t_2 t_1 + \frac{bt_2^2 t_1}{2} - \frac{bt_1^3}{6} + \frac{\theta t_2 t_1^3}{6} - \frac{\theta t_2^3 t_1}{3} + \frac{\theta t_1^4}{24} - \frac{b\theta t_1^5}{20} \right] \dots(1.13)$$

Total amount of deteriorated units is given by

$$I_D = \int_0^{t_1} z(t) I(t) dt + \int_{t_1}^{t_2} z(t) I(t) dt$$

$$I_D = a\theta \left[(r-1) \left(\frac{t_1^3}{3} - \frac{bt_1^4}{8} \right) + \frac{t_2^3}{6} - \frac{bt_2^4}{8} - \frac{t_2 t_1^2}{2} + \frac{t_1^3}{3} + \frac{bt_2^2 t_1^2}{4} + \frac{bt_1^4}{8} \right]$$

$$I_D = a\theta \left[r \left(\frac{t_1^3}{3} - \frac{bt_1^4}{8} \right) + \frac{t_2^3}{6} - \frac{bt_2^4}{8} - \frac{t_2 t_1^2}{2} + \frac{bt_2^2 t_1^2}{4} \right] \dots(1.14)$$

$$I_s = - \int_{t_2}^{t_3} I(t) dt - \int_{t_3}^T I(t) dt$$

$$= - \left[\int_{t_2}^{t_3} \frac{\delta a}{b} [e^{-bt} - e^{-bt_2}] dt + \int_{t_3}^T \frac{a(r-\delta)}{b} (e^{-bT} - e^{-bt}) dt \right]$$

$$= - \left[\frac{\delta a}{b} \left\{ (t_2 - t_3) e^{-bt_2} + \frac{e^{-bt_2} - e^{-bt_3}}{b} \right\} \right. \\ \left. + \frac{a(r-\delta)}{b} \left\{ e^{-bT} (T - t_3) + \frac{e^{-bt_2} - e^{-bt_3}}{b} \right\} \right] \dots(1.15)$$

Total amount of lost sales is given by

$$\begin{aligned}
 I_L &= \int_{t_2}^{t_3} (1-\delta)R(t)dt + \int_{t_3}^T (1-\delta)R(t)dt \\
 &= \int_{t_2}^T (1-\delta)R(t)dt \\
 &= \int_{t_2}^T (1-\delta)ae^{-bt}dt \\
 I_L &= \frac{(1-\delta)a}{b} [e^{-bt_2} - e^{-bT}]. \quad \dots(1.16)
 \end{aligned}$$

From equation (1.9) we have

$$I(t_1) = \left[a(r-1) \left\{ t_1 - \frac{bt_1^2}{2} - \frac{\theta t_1^3}{3} + \frac{b\theta t_1^4}{4} \right\} \right] \quad \dots(1.17)$$

From equation (1.10) we have

$$I(t_1) = a \left[(t_2 - t_1) - \frac{b}{2}(t_2^2 - t_1^2) - \frac{\theta t_2 t_1^2}{2} + \frac{b\theta t_2^2 t_1^2}{2} + \frac{\theta t_2^3}{3} + \frac{\theta t_1^3}{6} \right] \quad \dots(1.18)$$

By equation (1.17) and (1.18) we get

$$\begin{aligned}
 r \left[t_1 - \frac{bt_1^2}{2} - \frac{\theta t_1^3}{3} + \frac{b\theta t_1^4}{4} \right] &= \left[t_2 - \frac{b}{2}t_2^2 - \frac{\theta t_2 t_1^2}{2} \right. \\
 &\quad \left. + \frac{b\theta t_1^2 t_2^2}{2} - \frac{\theta t_1^3}{6} + \frac{\theta t_2^3}{3} + \frac{b\theta t_1^4}{4} \right] \quad \dots(1.19)
 \end{aligned}$$

Now we consider as

$$t_2 = f(t_1) \quad \dots(1.20)$$

From equation (1.11) we have

$$I(t_3) = \frac{\delta a}{b} [e^{-bt_3} - e^{-bt_2}] \quad \dots(1.21)$$

From equation (1.12) we have

$$I(t_3) = \frac{a(r-\delta)}{b} \quad \dots(1.22)$$

On comparing equations (1.21) and (1.22) we have

$$re^{-bt_3} = \delta e^{-bt_2} + (r - \delta)e^{-bT}$$

Now we are consider as

$$t_3 = g(T, t_1) \quad \dots(1.23)$$

Hence, the total average cost of the system per unit time is given by

$$C = \frac{1}{T} [C' + C_H I_H + C_D \cdot I_D + C_s I_s + C_L I_L] = \frac{R}{T} \quad \dots(1.24)$$

$$C = \frac{1}{T} \left[C' + C_H a \left\{ r \left(\frac{t_1^2}{2} - \frac{bt_1^3}{6} - \frac{\theta t_1^4}{12} + \frac{b\theta t_1^5}{20} \right) + \frac{[f(t_1)]^2}{2} - \frac{b[f(t_1)]^3}{6} + \frac{5\theta[f(t_1)]^4}{24} + b\theta[f(t_1)]^5 - [f(t_1)]t_1 + \frac{b[f(t_1)]^2 t_1}{2} - \frac{bt_1^3}{6} + \frac{\theta t_1}{24} - \frac{b\theta t_1^3}{20} \right\} + C_D a \theta \left\{ r \left(\frac{t_1^3}{3} - \frac{bt_1^4}{8} \right) + \frac{[f(t_1)]^3}{6} - \frac{b[f(t_1)]^4}{8} - \frac{f(t_1)t_1}{2} + \frac{b[f(t_1)]^2 t_1^2}{4} \right\} - C_s \left\{ \frac{\delta a}{b} \left((f(t_1) - g(T, t_1))e^{-bf(t_1)} + \frac{e^{-bf(t_1)} - e^{-bf(T, t_1)}}{b} \right) + \frac{a(r - \delta)}{b} \left(\frac{e^{-bf(t_1)} - e^{-bg(T, t_1)}}{b} + (T - g(T, t_1))e^{-bT} \right) \right\} + C_L \frac{(1 - \delta)}{b} a \{ e^{-bf(t_1)} - e^{-bT} \} \right] = \frac{R}{T} \quad \dots(1.25)$$

CONCLUSIONS

An order level inventory model for deteriorating items is developed. The demand is taken as exponential function of time and production rate as taken to be dependent on demand function. The deteriorating of unit in an inventory system is taken to be time dependent. Shortages are allowed and partially backlogged.

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